Many-Objective Nurse Scheduling Using NSGA-II based on Pareto Partial Dominance with Linear Subset-Size Scheduling

Makoto OHKI @ Tottori University, Japan

Unconstrained Many-Objective Nurse Scheduling

	d_1	d_2	d_3	⋯	d_j	⋯	d_D
n_1	D	N	E	⋯	E	⋯ H	T
n_2	N	H	D	⋯	E	⋯ M	R
⋮	⋮				⋮		⋮
n_i	R	R	E	⋯	N	⋯ M	D
⋮	⋮				⋮		⋮
n_N	D	D	H	⋯	H	⋯ M	R

Nurse Schedule (individual)

* Twelve penalties are defined to evaluate the schedule. (see the paper for detail)

* Twelve objective functions are defined by means of the penalties p_{ik}.

$$f_k = \begin{cases} 1\Big/\left(1+\sum_{i=1}^{N} p_{ki}\right), & k=1,2,\cdots,6 \\ 1\Big/\left(1+\sum_{i=1}^{D} p_{ki}\right), & k=7,8,\cdots,12 \end{cases}$$

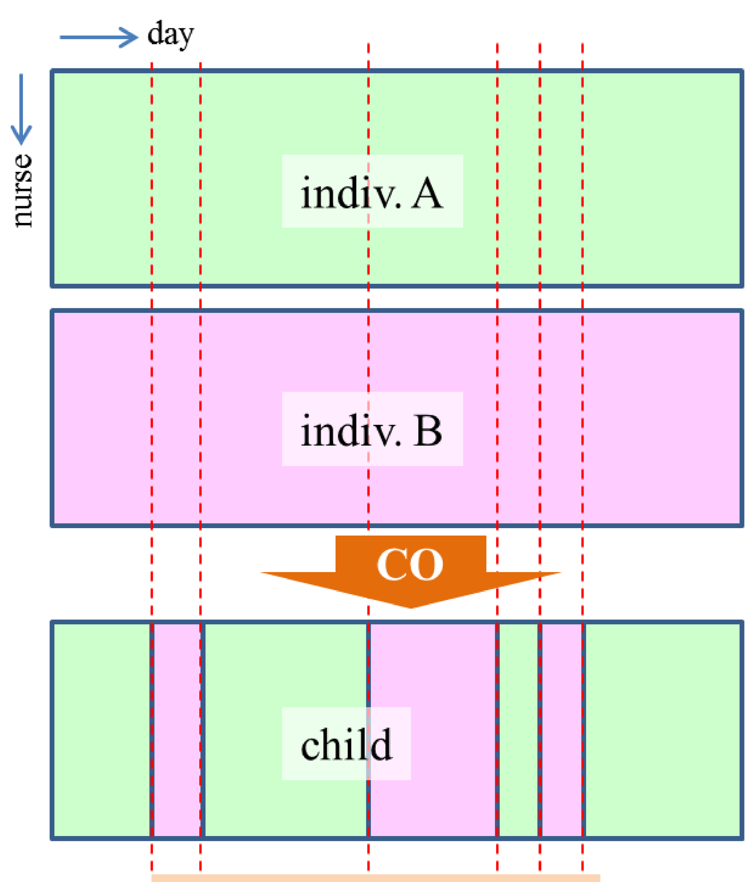
* The problem is formulated as cMOP.

$$\begin{cases} \max. & \mathbf{f}(\mathbf{x}) = \left[f_1(\mathbf{x}), f_2(\mathbf{x}), \cdots, f_{12}(\mathbf{x}) \right]^T \\ \text{s.t.} & \mathbf{x} \in \mathbf{S} \equiv \{\text{feasible space}\} \end{cases}$$

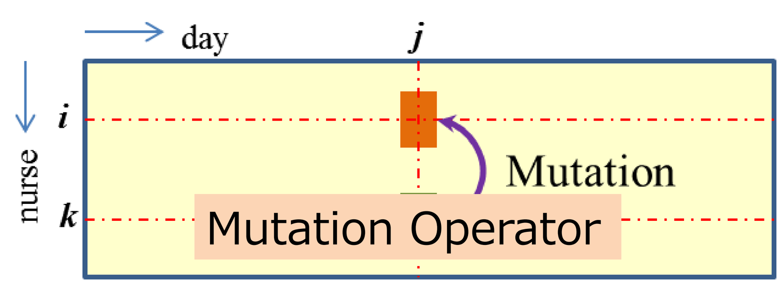
Binomial relation that point $\mathbf{f(x)}$ dominates point $\mathbf{f(y)}$

$$\mathbf{f}(\mathbf{x}) \succ \mathbf{f}(\mathbf{y}) \triangleq \forall i \in \mathbf{M}: f_i(\mathbf{x}) \geq f_i(\mathbf{y}) \wedge \exists i \in \mathbf{M}: f_i(\mathbf{x}) > f_i(\mathbf{y}), \quad \mathbf{M} = \{1,2,\cdots,12\}$$

* The initialization and the following operators release the problem constraints.



Crossover Operator



Mutation Operator

* By means of these operators, the optimization problem can be unconstrained.

$$\max. \quad \mathbf{f}(\mathbf{x}) = \left[f_1(\mathbf{x}), f_2(\mathbf{x}), \cdots, f_{12}(\mathbf{x}) \right]^T$$

* Although a framework of NSGA-II can be applied to this problem, too many objective functions. ⇒ uMaOP

Improvement of MSGA-II based on Pareto Partial Dominance

Pareto Partial Dominance (PPD) that makes it easier to determine the superiority/inferiority relationships between individuals [Sato2010].

$$\mathbf{f}(\mathbf{x}) \triangleright \mathbf{f}(\mathbf{y}) \triangleq \forall i \in \mathbf{R} \subset \mathbf{M}: f_i(\mathbf{x}) \geq f_i(\mathbf{y}) \wedge \exists i \in \mathbf{R} \subset \mathbf{M}: f_i(\mathbf{x}) > f_i(\mathbf{y})$$

Sato's Algorithm has several problems.

* The subset size of the objective functions to be used fore Pareto partial dominance is required to specify before the optimization in a form of a combination list.
* An appropriate value of the subset size according to the complexity of the problem is unknown.

The contents of the combination list greatly influence the result. Furthermore, the creation of the combination list requires the user to have a high skill.

* NSGA-II-PPD performs ND sorting for all objective functions at a specific generation cycle, and preserves parents as an archive set **A** for the next generation. This process is able to generate child individuals having the same contents as the already existing individual in **A**. As a result, the same individuals increases in the first front set.

Improvement#1: Subset Size Scheduling (SSS)

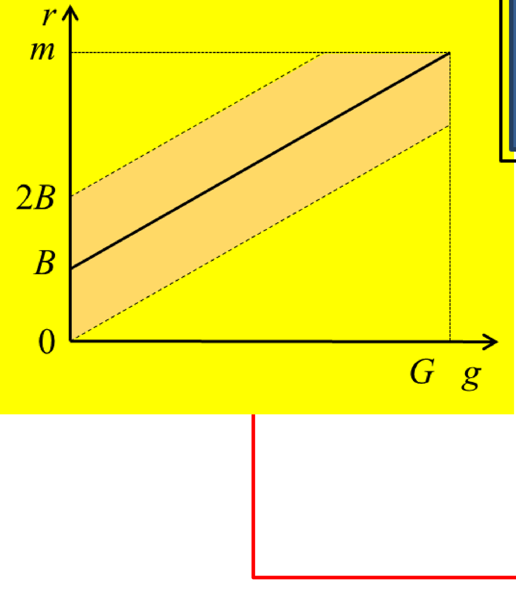
* The combination list is eliminated from the algorithm.
* SSS is improved. The parameter r is given by the following equations.

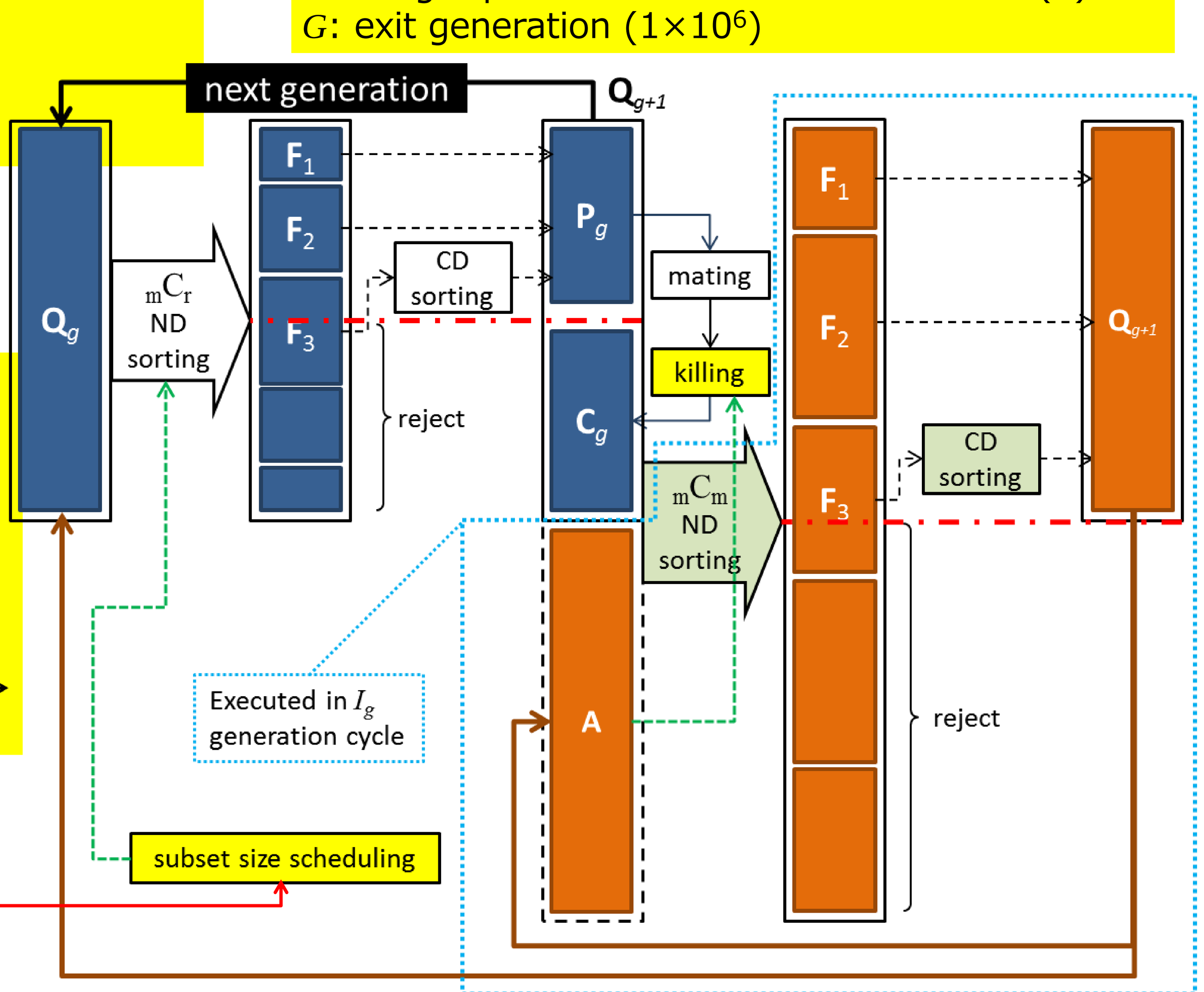
Improvement#2: Killing Unwanted Individuals Given by Mating

* Compare each individual in $\mathbf{C}_g$ created by the mating and all individuals in **A**.
* If it is an individual with the same content, kill it.
* This can be easily done by setting the value of all objective functions of such the individual to 0.

$$\begin{cases} q = \dfrac{g \cdot m}{G} + \text{rand\_int}(2B+1) - B \\ r = \begin{cases} B, & q < B \\ q, & B \leq q < m \\ m, & q \geq m \end{cases} \end{cases}$$

m: number of objective functions (12)
rand_int(*): function returns a random integer<*.
B: integer parameter as $B \in \mathbf{N}$ and $1<B<m/2$ (2)
G: exit generation (1×10^6)

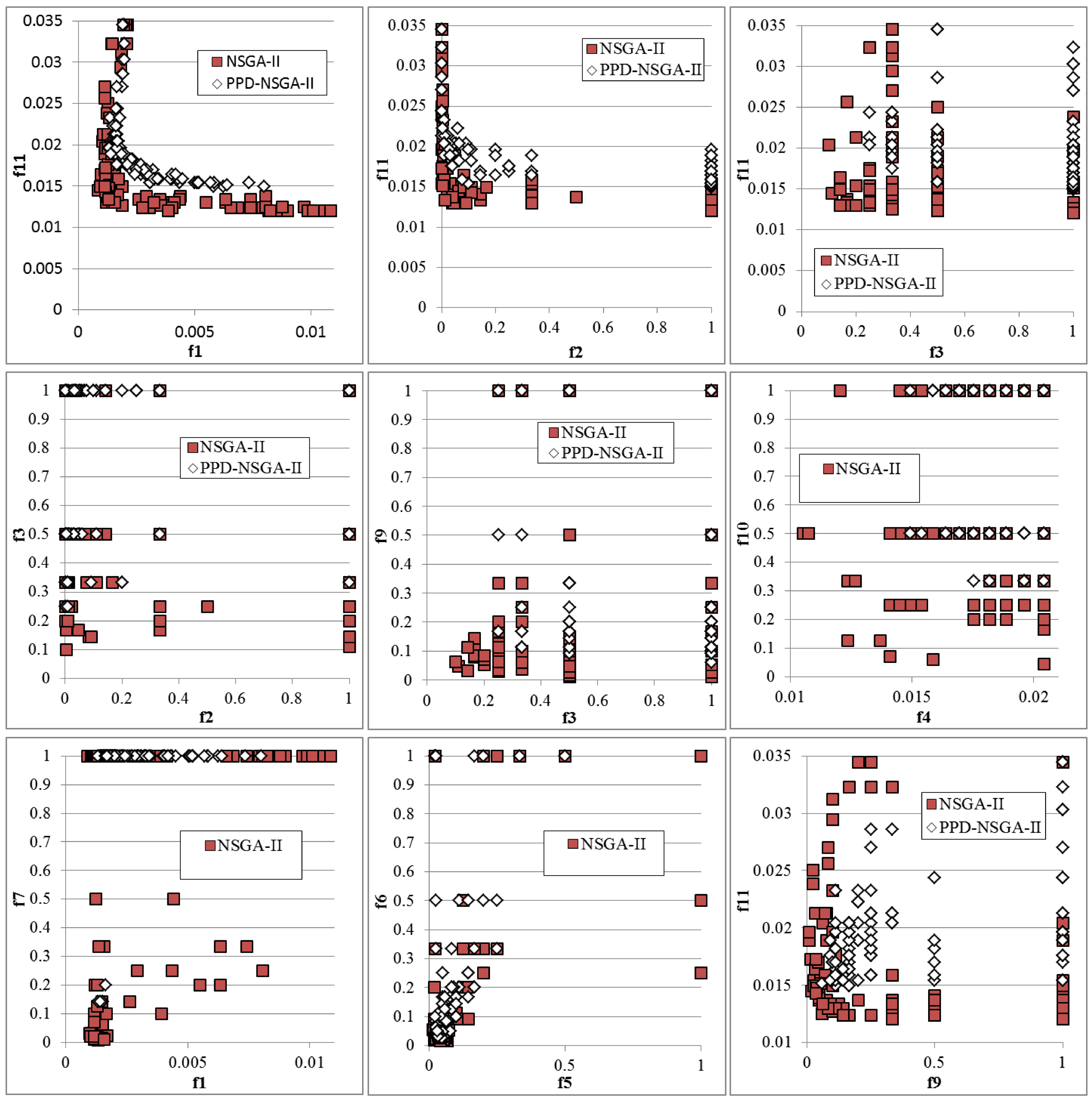




PPD-NSGA-II

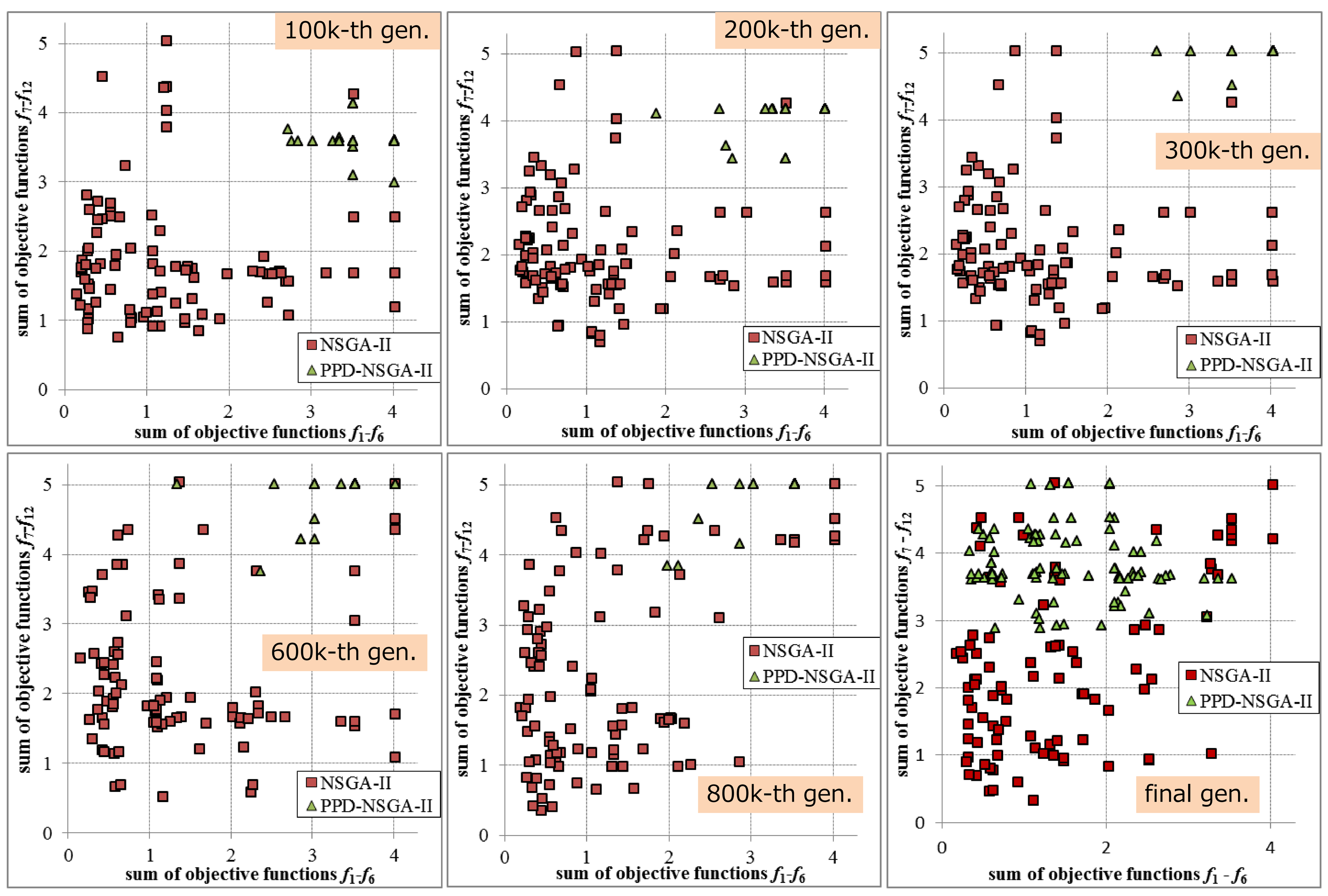
Nurse Scheduling in Actual Hospital

*** Distribution of the first front set (FFS) at the final generation expressed on biaxial planes sliced out from the hyper space with 12 objective functions.**



*** Distribution of $\mathcal{FFS}$ on s1-s2 plane with generation progression.**

$$s_1(\mathbf{x}) = \sum_{i=1}^{6} f_i(\mathbf{x}), \qquad s_2(\mathbf{x}) = \sum_{i=7}^{12} f_i(\mathbf{x})$$



*** Comparison of Norm and MS values of $\mathcal{FFS}$ at the final generation.**

$$\text{Norm} = \frac{1}{|\mathcal{FFS}|} \sum_{j=1}^{|\mathcal{FFS}|} \sqrt{\sum_{i=1}^{m} f_i(\mathbf{x}_j)^2}, \quad \text{MS} = \sqrt{\sum_{i=1}^{m} \left(\max_{j=1}^{|\mathcal{FFS}|} f_i(\mathbf{x}_j) - \max_{j=1}^{|\mathcal{FFS}|} f_i(\mathbf{x}_j) \right)^2}$$

	NSGA-II	PPD-NSGA-II
Norm	1.5589	2.1795
MS	2.9126	2.3970

Discussion

*** Distribution of $\mathcal{FFS}$ at final generation:**

Depending on the biaxial plane of interest, we have observed cases where PPD-NSGA-II (proposed) has been advantageous, NSGA-II (conventional) has been advantageous, and both superiority/inferiority has not been determined.

*** Distribution of $\mathcal{FFS}$ on s1-s2 plane with generation progression :**

FFS given by PPD-NSGA-II converges to POS faster than NSGA-II.
However, several solutions by PPD-NSGA-II have been lost at the final generation.
Since the observation has been made on the S2-S1 plane, the lost solutions are not necessarily good, but it is considered that some degradation has occurred.

*** Comparison of Norm and MS values of $\mathcal{FFS}$ at the final generation:**

For convergence to POS (Norm), PPD-NSGA-II is advantageous, and the spread of the solution set is almost equivalent.

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