Linear Subset Size Scheduling for Many-Objective Optimization Using NSGA-II Based on Pareto Partial Dominance

Makoto OHKI, Tottori University, Japan

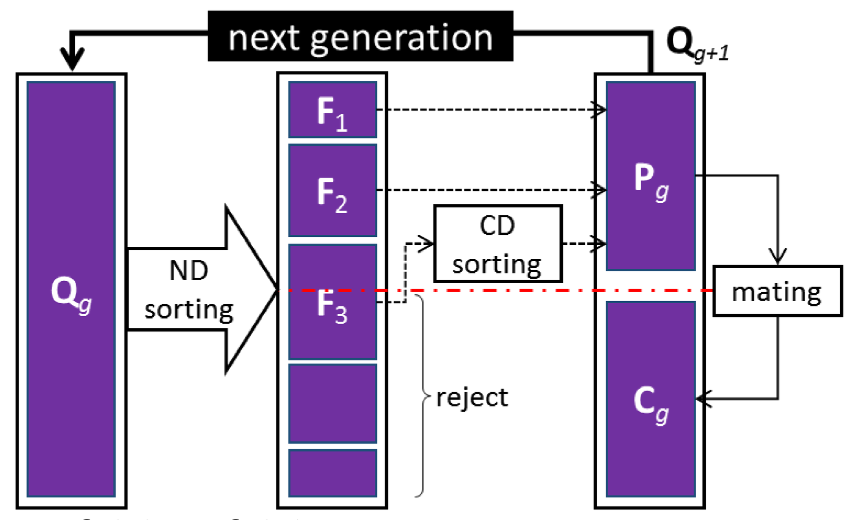
Introduction

● Multi-Objective Optimization Problem (MOP) :

$$\begin{cases} \max. & \mathbf{f}(\mathbf{x}) = \left[f_1(\mathbf{x}), f_2(\mathbf{x}), \cdots, f_m(\mathbf{x}) \right]^T \\ \text{s.t.} & \mathbf{x} \in \mathbf{S} = \{\text{feasible solution space}\} \end{cases}$$

When the following relation is satisfied, $\mathbf{f}(\mathbf{x})$ dominates $\mathbf{f}(\mathbf{y})$.

$$\mathbf{f}(\mathbf{x}) \succ \mathbf{f}(\mathbf{y}) \triangleq \forall i \in \mathbf{M} : f_i(\mathbf{x}) \geq f_i(\mathbf{y}) \wedge \exists i \in \mathbf{M} : f_i(\mathbf{x}) > f_i(\mathbf{y})$$



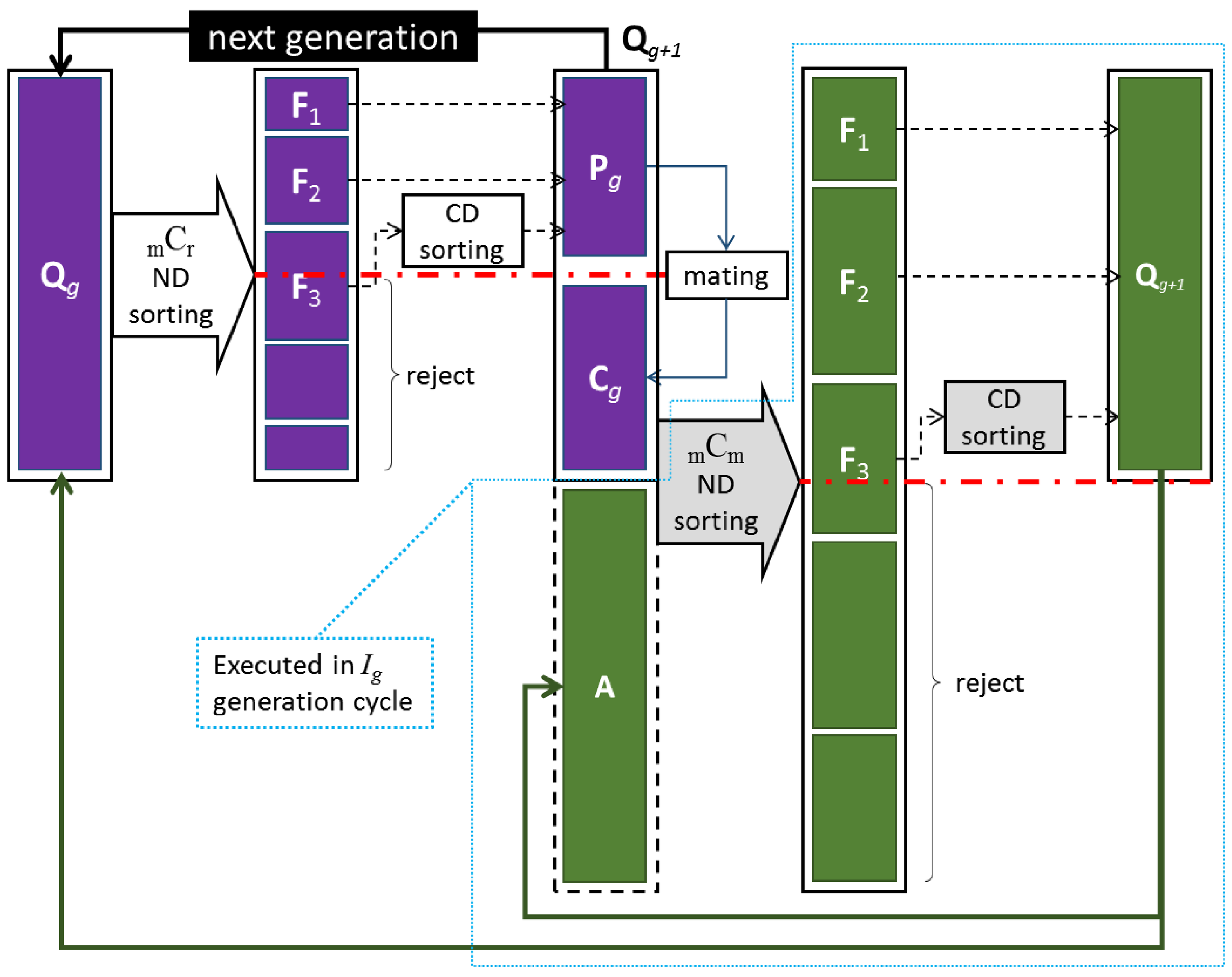
MOP is a problem to find a set of non-dominated solutions in S as shown by the following formula. NSGA-II [Deb2000] with non-dominated sorting (ND) is known as a powerful framework for MOP.

$$\mathcal{POS} = \{ \mathbf{x} \in \mathbf{S} \mid \neg \exists \mathbf{y} \in \mathbf{S}. \mathbf{f}(\mathbf{y}) \succ \mathbf{f}(\mathbf{x}) \}$$

● For MOP where the number of objective functions is 4 or more (MaOP [Zitzler1998]), since NSGA-II performs ranking selection with ND for all m objective functions, most solutions become non-inferior solutions. Therefore, the superiority/inferiority of the solution within the population is difficult to determined [Tsuchida2009].

● Sato et al. have proposed Pareto partial dominance that makes it easier to determine the superiority/inferiority relationship between solutions.

$$\mathbf{f}(\mathbf{x}) \rhd \mathbf{f}(\mathbf{y}) \triangleq \forall i \in \mathbf{R} \subset \mathbf{M} : f_i(\mathbf{x}) \geq f_i(\mathbf{y}) \wedge \exists i \in \mathbf{R} \subset \mathbf{M} : f_i(\mathbf{x}) > f_i(\mathbf{y})$$



Improvement of NSGA-II Based on Pareto Partial Dominance

Problems of NSGA-II based on Pareto Partial Dominance (NSGA-II-PPD),

* The subset size of the objective functions to be used fore Pareto partial dominance is required to specify before the optimization in a form of a combination list.

* An appropriate value of the subset size according to the complexity of the problem is unknown.

The contents of the combination list greatly influence the result. Furthermore, the creation of the combination list requires the user to have a high skill.

* NSGA-II-PPD performs ND sorting for all objective functions at a specific generation cycle, and preserves parents as an archive set **A** for the next generation. This process is able to generate child individuals having the same contents as the already existing individual in **A**. As a result, the same individuals increases in the first front set.

Improvement #1: Subset Size Scheduling (SSS)

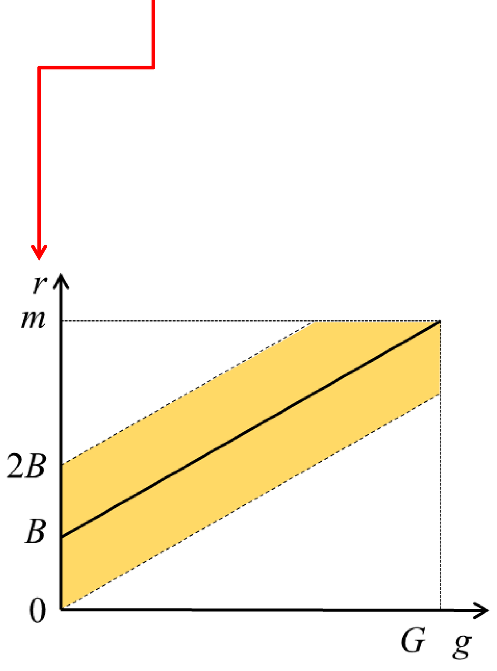
* The combination list is eliminated from the algorithm.

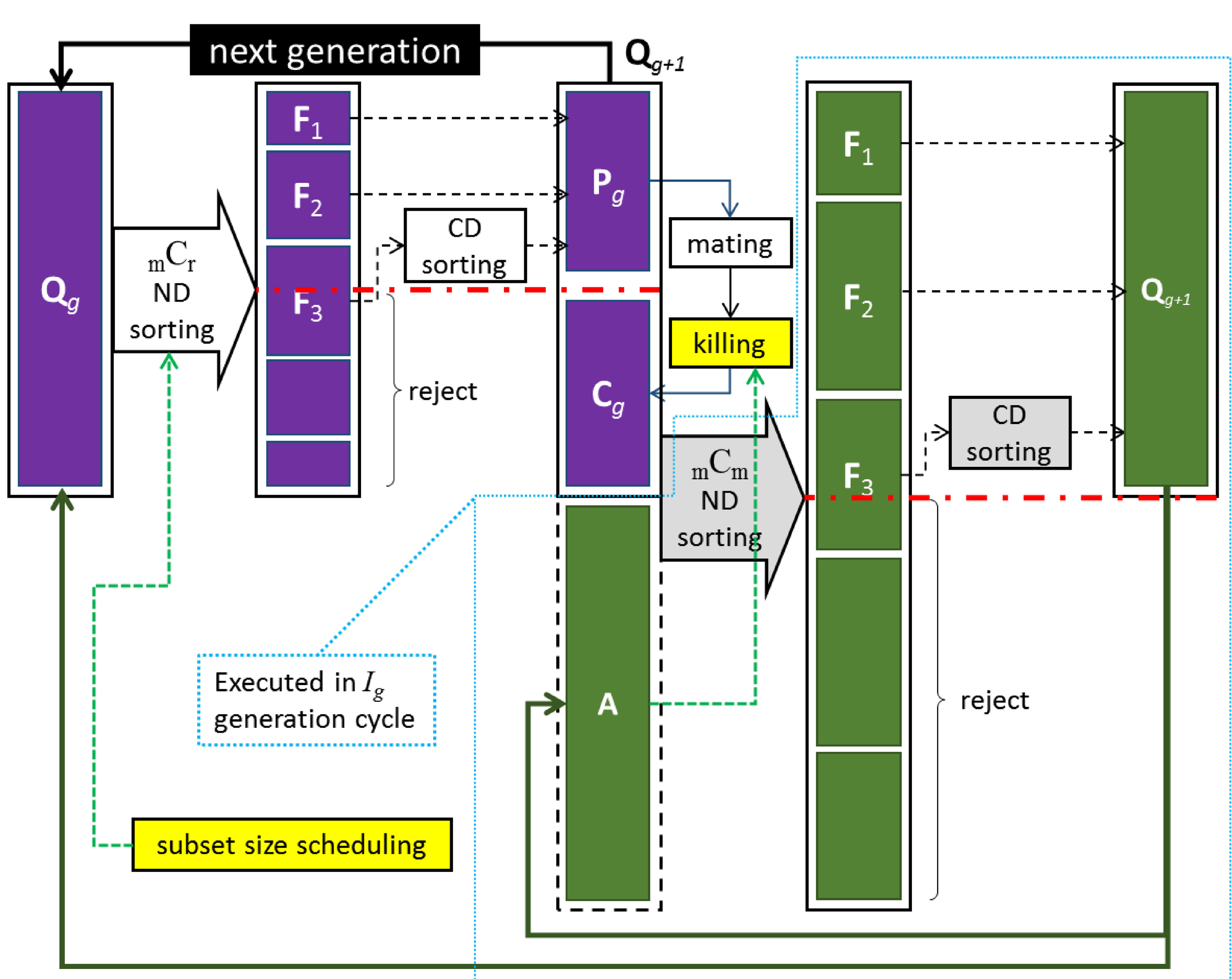
* SSS is improved. The parameter r is given by the following equations.

$$q = \frac{g \cdot m}{G} + \text{rand\_int}(2B+1) - B$$

$$r = \begin{cases} B, & q < B \\ q, & B \leq q < m \\ m, & q \geq m \end{cases}$$

m: number of objective functions
rand_int(*): function returns a random integer<*.
B: integer parameter as $B \in \mathbf{N}$ and $1<B<m/2$
G: exit generation





Improvement #2: Killing unwanted individuals

* Compare each individual in $\mathbf{C}_g$ created by the mating and all individuals in **A**.

* If it is an individual with the same content, kill it.

* This can be easily done by setting the value of all objective functions of such the individual to 0.

Verification by Many-Objective 0/1 KnapSack Problem

Many-objective 0/1 KSP is defined by n-dimensional vecitor $\mathbf{x} \in \{0,1\}^n$ and the following formulas [Zitzler1999].

$$\begin{cases} \max. & \mathbf{f}(\mathbf{x}) = \left[f_1(\mathbf{x}), f_2(\mathbf{x}), \cdots, f_m(\mathbf{x}) \right]^T \\ \text{s.t.} & \sum_{j=1}^{n} w_{ij} \cdot x_j \leq c_i \end{cases} \qquad f_i(\mathbf{x}) = \sum_{j=1}^{n} p_{ij} \cdot x_j \quad \text{for } i = 1, 2, \cdots, m$$

* POS obtained by the experimental optimization is evaluated by Norm[Sato2006] and Maximum Spread (MS) [Zitzler1999].

degree of convergence to $\mathcal{POS}$

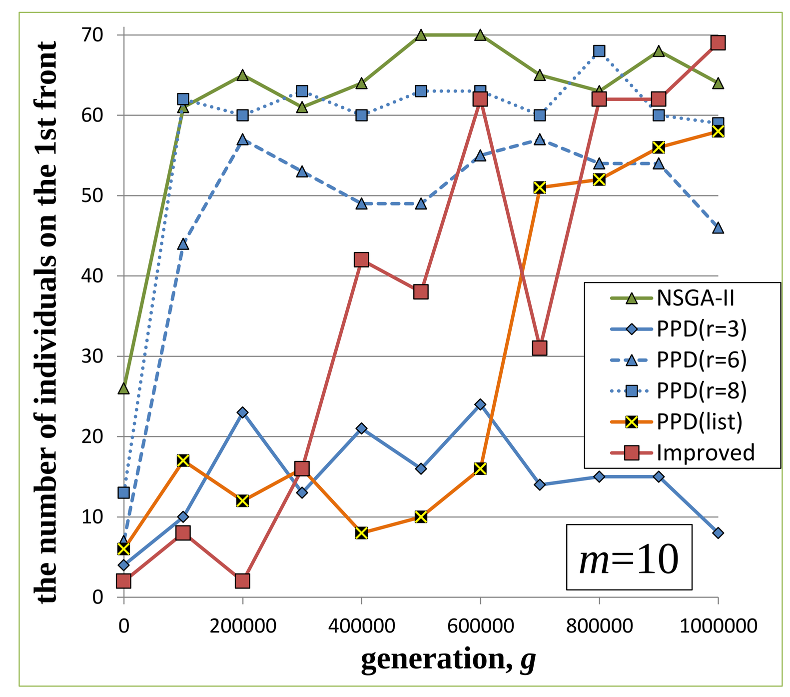
$$\text{Norm} = \frac{1}{|\mathcal{FFS}|} \sum_{j=1}^{|\mathcal{FFS}|} \sqrt{\sum_{i=1}^{m} f_i(\mathbf{x}_j)^2}$$

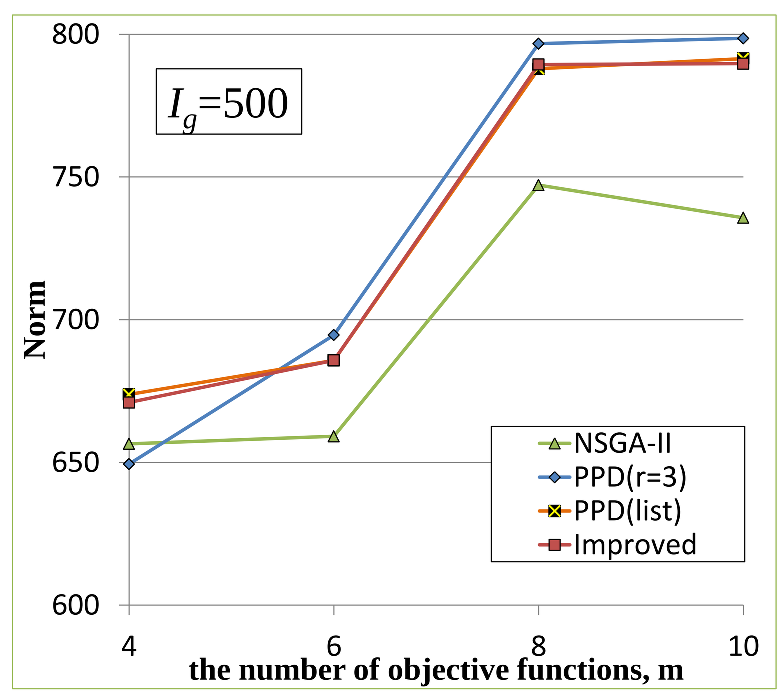
degree of spread of $\mathcal{FFS}$

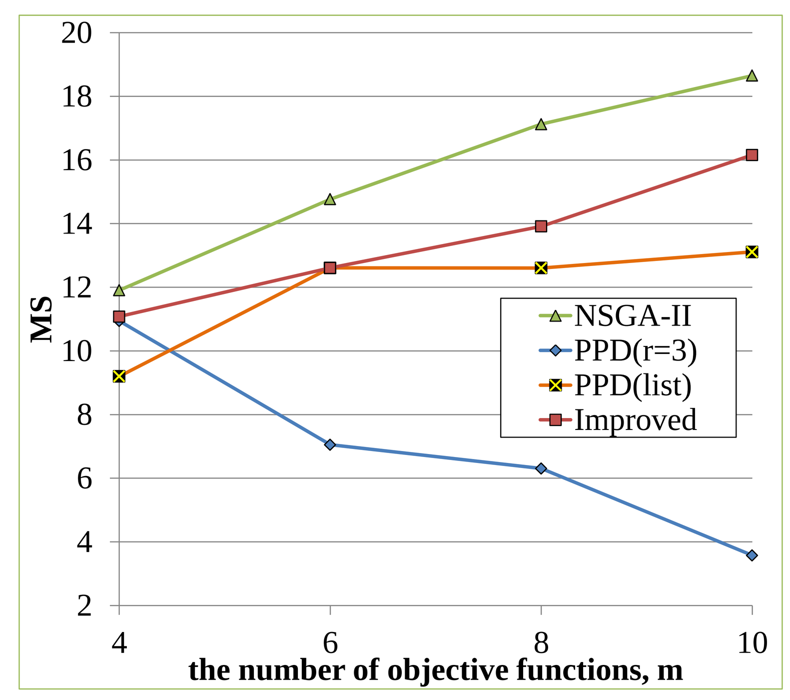
$$\text{MS} = \sqrt{\sum_{i=1}^{m} \left(\max_{j=1}^{|\mathcal{FFS}|} f_i(\mathbf{x}_j) - \min_{j=1}^{|\mathcal{FFS}|} f_i(\mathbf{x}_j) \right)^2}$$

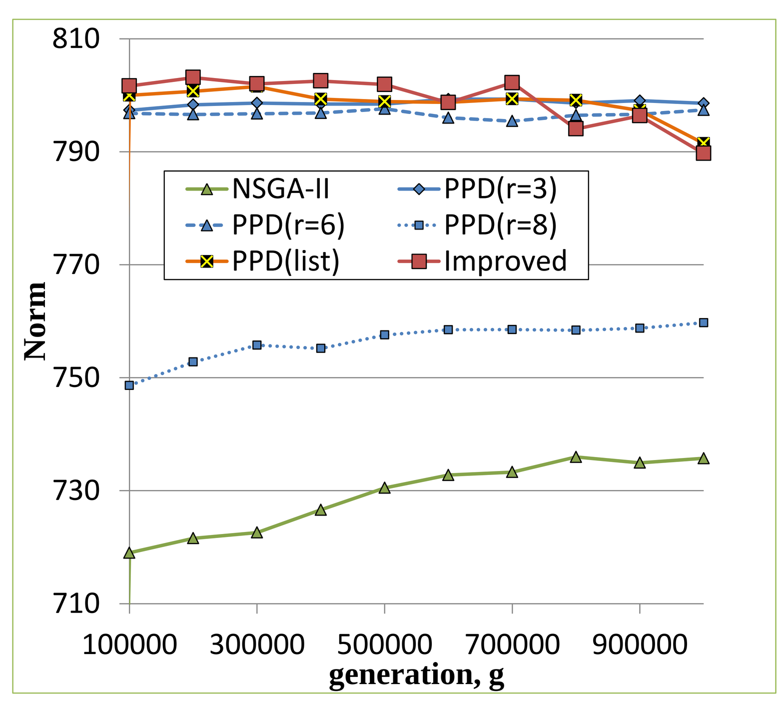
Table 1. Combination list for NSGA-II-PPD

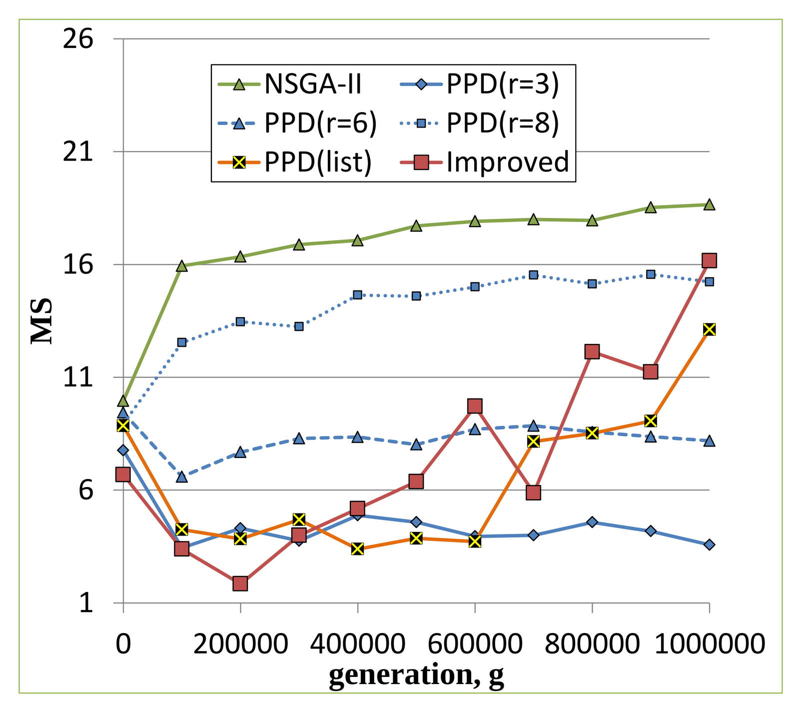
	generation range			
	0-500k	500k-900k	900k-1M	
m	r			
4	2	3	4	
6	3	5	6	
	generation range			
	0-300k	300k-600k	600k-900k	900k-1M
m	r			
8	3	5	7	8
10	3	6	8	10











Discussion and Conclusion

*FFS-g: In the improved technique, as the optimization progresses, the size of the first front gradually increases, finally becoming the largest.

*Norm-m: The improved technique gives the almost equivalent high convergence to $\mathcal{POS}$ as the others except for the conventional NSGA-II.

*MS-m: The improved technique gives relatively better diversity of $\mathcal{FFS}$.

*Norm-g: The improved technique gives almost the equivalent high convergence to $\mathcal{POS}$ as the others except for the conventional NSGA-II and NSGA-II-PPD(r=8).

*MS-g: In the improved technique, the diversity of $\mathcal{FFS}$ gradually increased, eventually gave the almost equivalent diversity as the best result.

● Overall, the improved method finally has the largest diversity of FFS and high convergence to POS.

In this paper, the improvement of NSGA-II-PPD has been proposed with the aim of improving the searching performance of MOEA for MaOP.

- Simple Subset Size Scheduling (SSS) of the number r of the objective functions for Pareto partial dominance.
- Killing the individual of the children $\mathbf{C}_g$ which has the same contents as the individual in the archive set **A**.

-------------Future Work-------------------

- Since the improved technique still has given insufficient results in terms of the diversity, we need to improve this point while maintaining the current convergence.
- We need to apply the improved technique to more complex problems.

15th International Conf. on Informatice in Control, Automatation and Robotices (ICINCO 2018), Porto, Portugal, July, 29-31, 2018