IEEE-IS'06, Sep.04-06, 2006, University of Westminster, London, UK

Nurse Scheduling by Using Cooperative GA with Efficient Mutation and Mountain-Climbing Operators

	1 (mon)	2 (tue)	3 (wed)	---------	30 (tue)
nurse1	D	S	D	---------	H
nurse2	D	D	N	---------	N
nurse3	k	M	h	---------	h
⋮	⋮	⋮	⋮		⋮
nurseN	D	N	k	---------	D

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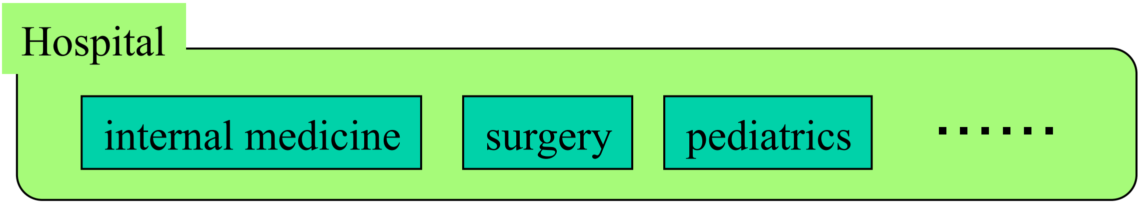
1. Introduction

●Generally a hospital is divided into some sections.



A nurse belongs to one of the sections. → 15－30 nurses / section

●A manager of the section or a chief nurse must make their schedule of one month.

●The nurse works on **three-shift system** in Japan,

09:00-17:00 **D**ay time duty,
17:00-01:00 **S**emi night duty
01:00-09:00 mid **N**ight duty.

As a duty, **M**eeting, **T**raining, **H**oliday, and **R**equested holiday are also considered.

- The chief nurse must consider a lot of requirements to make the schedule.

  for example,...
  - the number of the nurses at each duty shift
  - fairness of assignment of day and night duties
  - nursing quality
  - affinity between nurses at mid night duty
  - ・・・・・・・・・・・

- In our investigation, it takes 1.5－2 weeks to make the nurse schedule.

  → Nurse Scheduling is very difficult task.

To improve the difficulty, we propose an effective algorithm to **create** and **optimize** the nurse schedule by using **the cooperative GA**.

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2. Overview of Nurse Scheduling — Requirements

- A lot of **requirements** are summarized into 12 penalty functions (4 groups).

A. Six Requirements on Duty Pattern of Each Nurse n_i

F_{1i} : **duty load** of a nurse n_i of one month

$$F_{1i} = \sum_{j=1}^{D-1} p_{ij}$$: p_{ij} penalty value of nurse n_i at date d_j

nurse n_i

D	D	D	D	N	H	D	N	S	M	M	D	N	H	D	D	T	D	M	S	R	R	R	R	R	D	D	D	D	M

Table 1. Penalty values for duty pattern of **three consecutive days**.

	d_{ij}	duty pattern					
desirable pattern	0	DDD	DDH	DDN	DHD	DHH	DHN
		DNS	HDD	HDS	HHD	HHH	HNS
		SHH					
acceptable pattern	1	DDS	DSH	DNH	HDH	HDS	HHS
		HSH	SHD	NHD	NHH	NSH	
compromised pattern	2	DSD	DSN	DND	DNN	HSD	HSN
		SSH	NDH	NDS	NHS	NNH	
prohibited pattern	5	DSD	DSN	DND	DNN	HSD	HSN
		HND	HNN	SDD	SDH	SDS	SDN
		SNS	SSD	SSS	SSN	SND	SNH
		SNS	SNN	NDD	NDN	NHN	NSD
		NSS	NSN	NND	NNS	NNN	

- Assignment of duties should be **fair**.

F_{2i} : Fairness of assignment of day time duties

$$F_{2i} = \left| N_i^{\mathrm{hom}} - N_{\mathrm{hom}} \right|$$

N_i^{hom} → the number of holidays of the nurse n_i of this month.

F_{3i} : Fairness of assignment of night duties

$$F_{2i} = \max\left(N_i^{sem} - N_{sem}, 0\right) + \max\left(N_i^{mid} - N_{mid}, 0\right)$$

N_i^{mid} → the number of mid night duties of the nurse n_i of this month.

N_i^{sem} → the number of semi night duties of the nurse n_i of this month.

- It is not undesirable to **intensively assign a night duty** to some nurses.

F_{4i} : Intensiveness of night duty

$$F_{4i} = \sum_{j=1}^{D-5} \max\left(N_{night/6}(i, j) - 3, 0\right)$$

$N_{night/6}(i, j)$ → the number of **night duties** for consecutive 6 days from duty date, d_j, of nurse , n_i.

● Some **duty patterns** are prohibited in some hospitals.

F_{5i} : Prohibited duty pattern

In this application, the mid night duty on the next from the training is prohibited.

$$F_{5j} := F_{5j} + 1$$

● A nurse should not work **successively more than 6 days**.

F_{6i} : Limit of consecutive duty days

$$F_{6j} := F_{6j} + \max\left(N_{cons} - 5, 0\right)$$

N_{cons}: the number of consecutive duty days

B. Six Requirements on Nurse Assignment at Duty Date d_j

- The **nursing level** at every duty time must be maintained.

F_{7j}: Achievement degree of **nursing level** at Day time duty

$$F_{7j} = \max\left(L_j^{day} - \sum_i L(n_i), 0\right) \quad for \quad n_i \in M_j^{day}$$

M_j^{day}: a set of nurses assigned to the Day time duty of date d_j

L_j^{day}: the lowest nursing level of Day time duty of date d_j

F_{8j}: Achievement degree of **nursing level** at Semi night duty

$$F_{8j} = \max\left(L_j^{sem} - \sum_i L(n_i), 0\right) \quad for \quad n_i \in M_j^{sem}$$

M_j^{sem}: a set of nurses assigned to the semi night duty of date d_j

L_j^{sem}: the lowest nursing level of semi night duty of date d_j

F_{9j}: Achievement degree of **nursing level** at mid Night duty

$$F_{9j} = \max\left(L_j^{mid} - \sum_i L(n_i), 0\right) \quad for \quad n_i \in M_j^{mid}$$

M_j^{mid}: a set of nurses assigned to the mid night duty of date d_j

L_j^{mid}: the lowest nursing level of mid night duty of date d_j

$L(n_i)$: $L(n_i)$ in ten steps

- The chief nurse defines **nursing level** of each nurse as shown in Table II.

F_{10j} : Affinity between nurses at mid night duty

Affinity between nurses affects the nursing level especially on mid night duty.

If one of bad affinity combinations, (n_3, n_{15}), (n_4, n_{18}) and (n_8, n_{20}), is found in the schedule, F_{10j} is increased with one penalty point.

$$F_{10\,j} := F_{10\,j} + 1$$

F_{11j} : Assigning tow or more new faces to mid night duty

$$F_{11\,j} := F_{11\,j} + \begin{cases} 0 & , \quad N_{j,new}^{mid} < 2 \\ \sum_{i=0}^{N_{j,new}^{mid}-2} \left(N_{j,new}^{mid} - i \right) & \quad N_{j,new}^{mid} \geq 2 \end{cases}$$

the number of new faces assigned to night duty of date d_j

The positions of a nurse, chief, head, expert, back-bone and new face, are defined in Table III.

F_{12j} : An expert or more skilled nurse must be assigned to day and mid night duty.

If an expert or more skilled nurse is **NOT** assigned to day and mid night duty, F_{12j} is increased with one penalty point.

$$F_{12\,j} := F_{12\,j} + 1$$

● These 12 requirements are classed into **four groups**.

partial penalty functions

$$G_1 = \sum_i \left(h_{11} F_{1i} + h_{12} F_{4i} + h_{13} F_{5i} \right)$$

$$G_2 = \sum_i \left(h_{21} F_{2i} + h_{22} F_{3i} + h_{33} F_{6i} \right)$$

$$G_3 = \sum_j \left(h_{31} F_{7j} + h_{32} F_{8j} + h_{33} F_{9j} \right)$$

$$G_4 = \sum_j \left(h_{41} F_{10j} + h_{42} F_{11j} + h_{43} F_{12j} \right)$$

variable penalty function

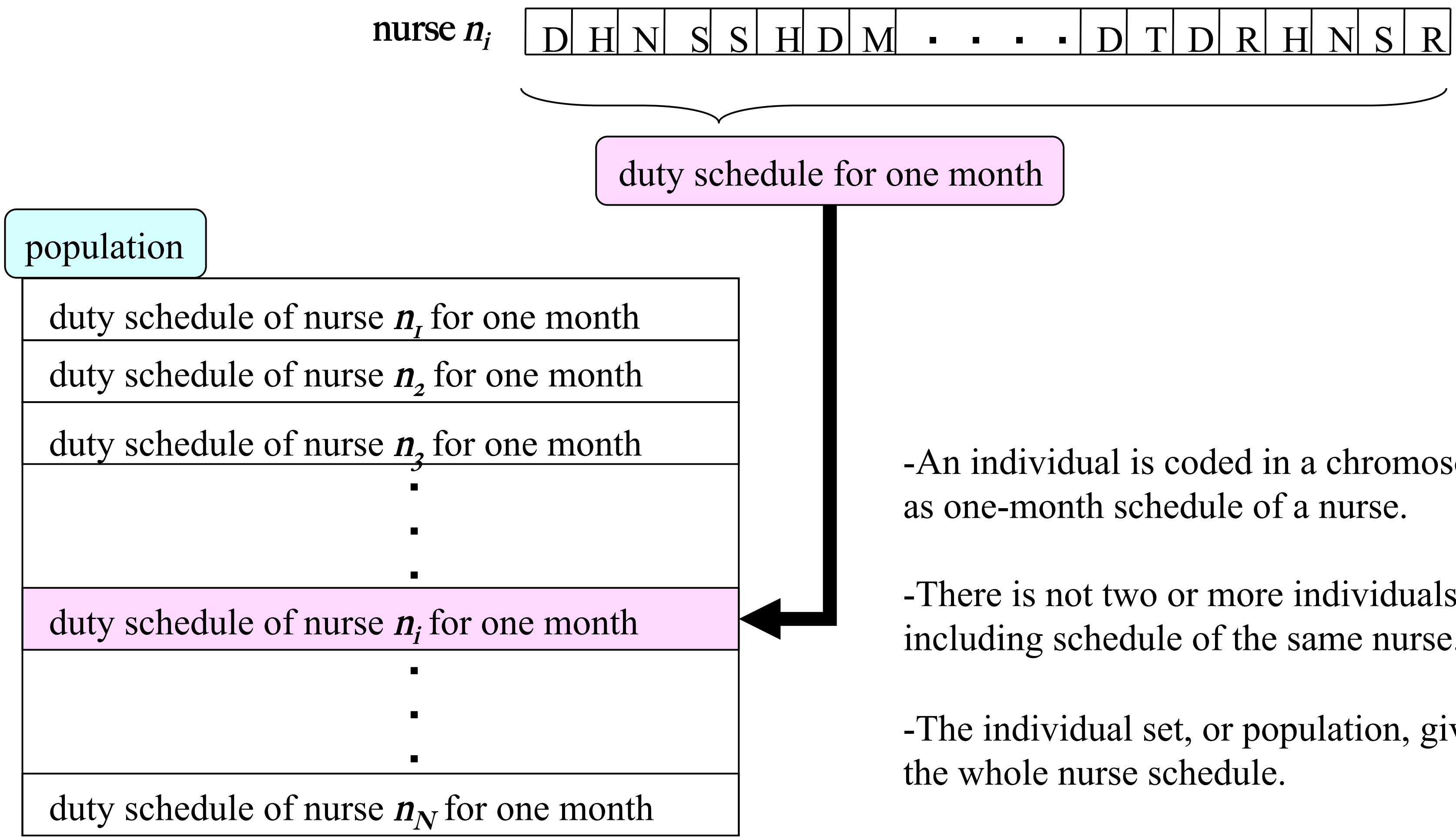
$$E_{\mathrm{var}}(\alpha) = \sum_k G_k^2$$

● The penalty coefficients h_{kl} are **modified** by the **mountain-climbing operator**.

A **constant penalty function** , E_{def}, with constant coefficients is also defined for preserving the **best population**.

3. Cooperative GA for Nurse Scheduling

A. Definition of Chromosome and Population

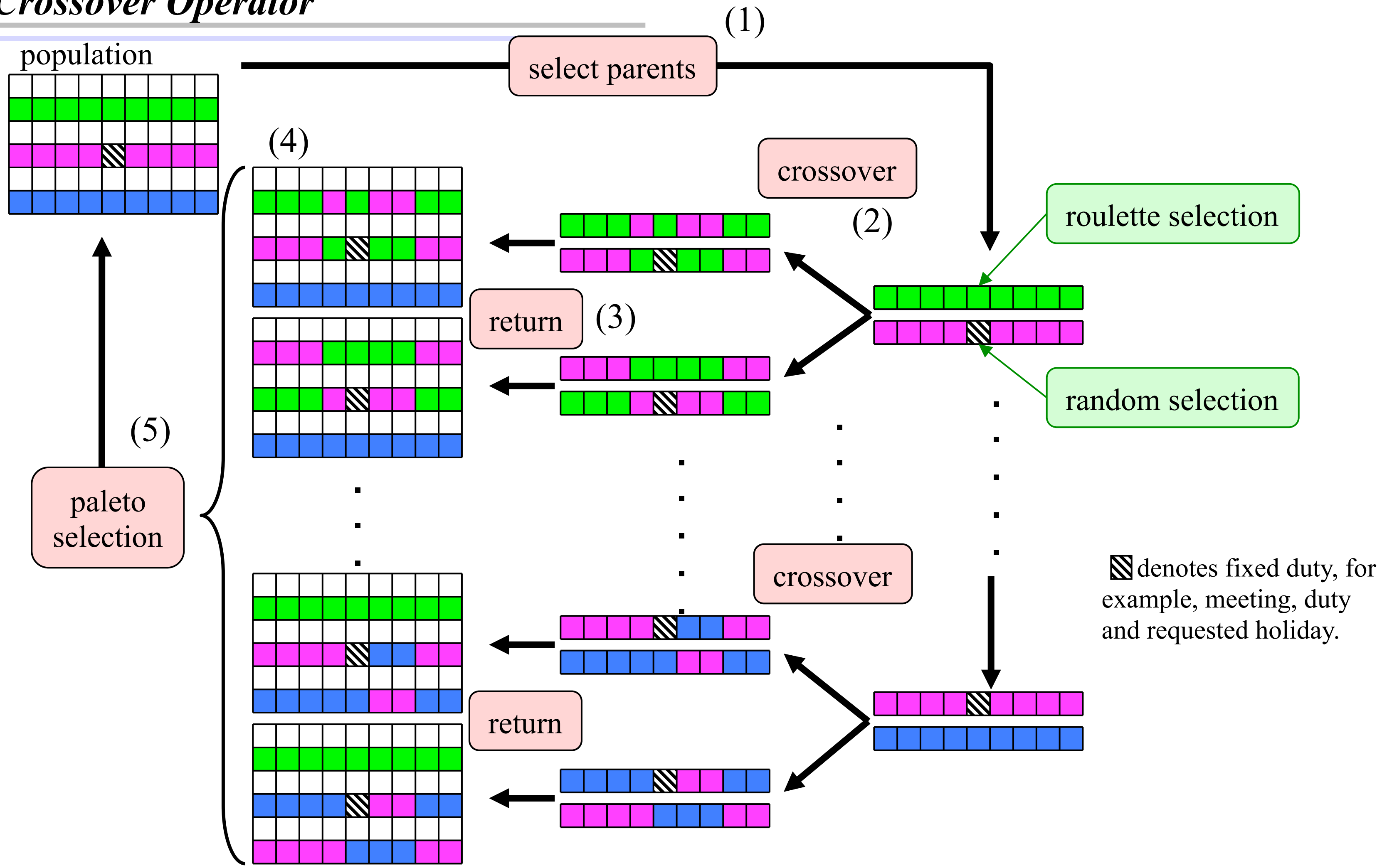


-An individual is coded in a chromosome as one-month schedule of a nurse.

-There is not two or more individuals including schedule of the same nurse.

-The individual set, or population, gives the whole nurse schedule.

B. Crossover Operator

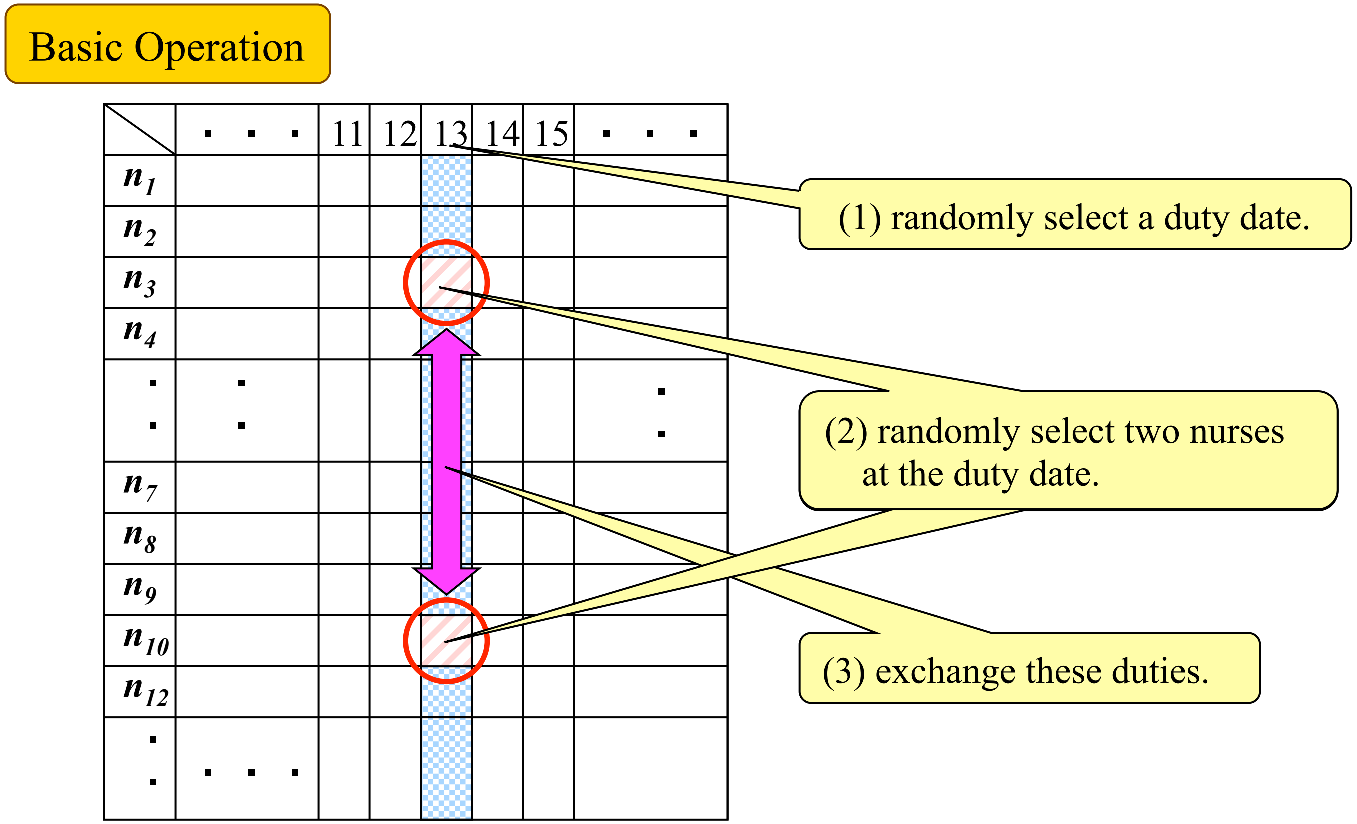


(1) Some pairs of the individual are selected from the population by using **roulette** and **random** selection manner.
(2) **Two-point crossover** is applied to these pairs. Then new pairs are created.
(3) These new pairs are **temporarily** returned to the population.
(4) These temporary populations are performed by the **variable penalty function** and the **constant penalty function**.
(5) A temporary population giving the smallest value of E_{var} is **preserved** to the next generation.

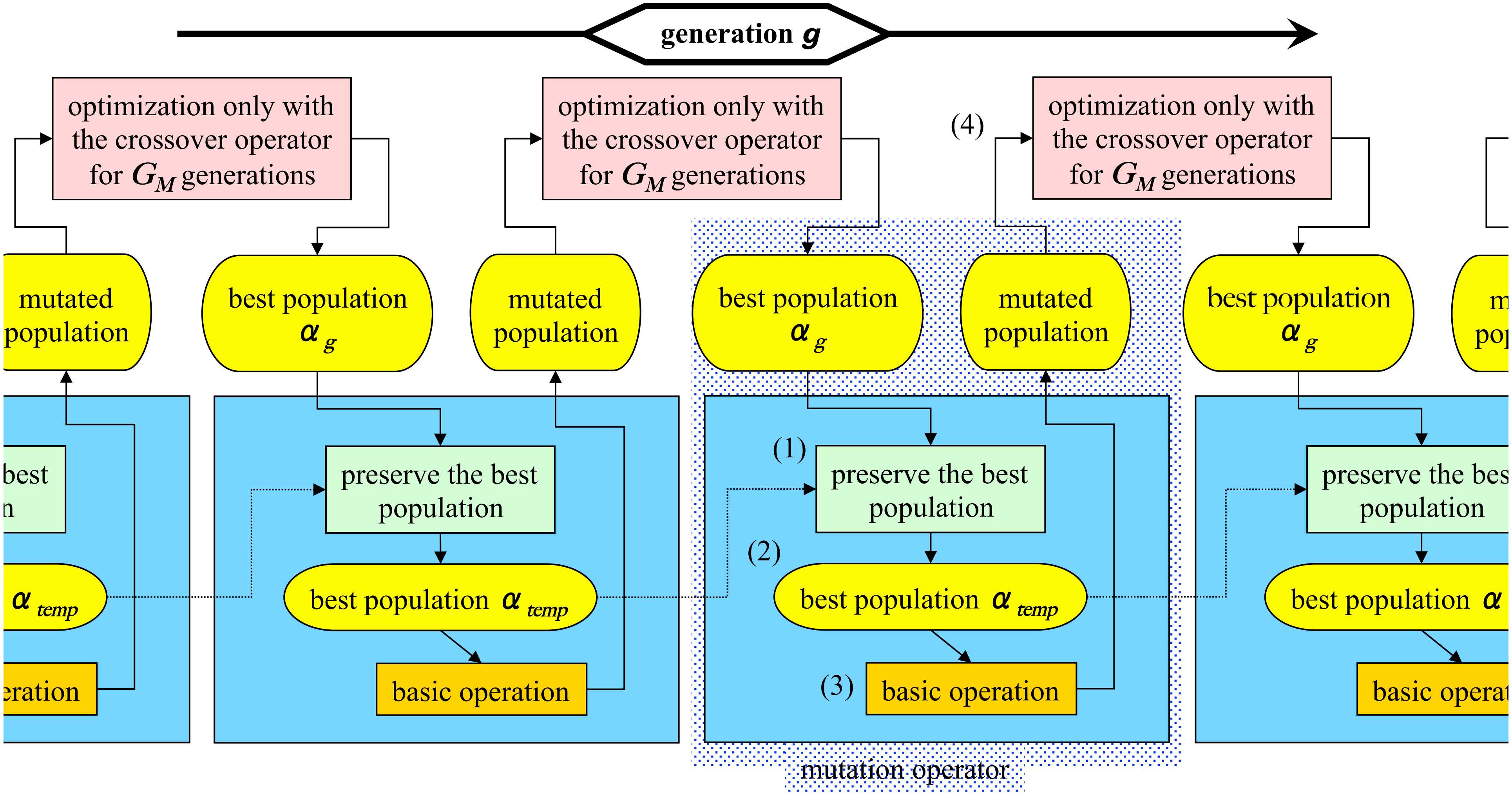
4. New Genetic Operators for the Cooperative GA

A. Mutation Operator

The aim of the **mutation** is to bring **small modification** into the population.
If a part of the population is modified by nonsense, the schedule become **meaningless**.
Besides, **recovering** such a modification is very **difficult**.



This basic operation is executed by a period for G_M generations.



(1) The mutation operator compares the population given by the G_M generations only with CO and the previous best population.

(2) The better population is selected as the best population at this generation.

(3) The basic mutation operation is applied to the selected population

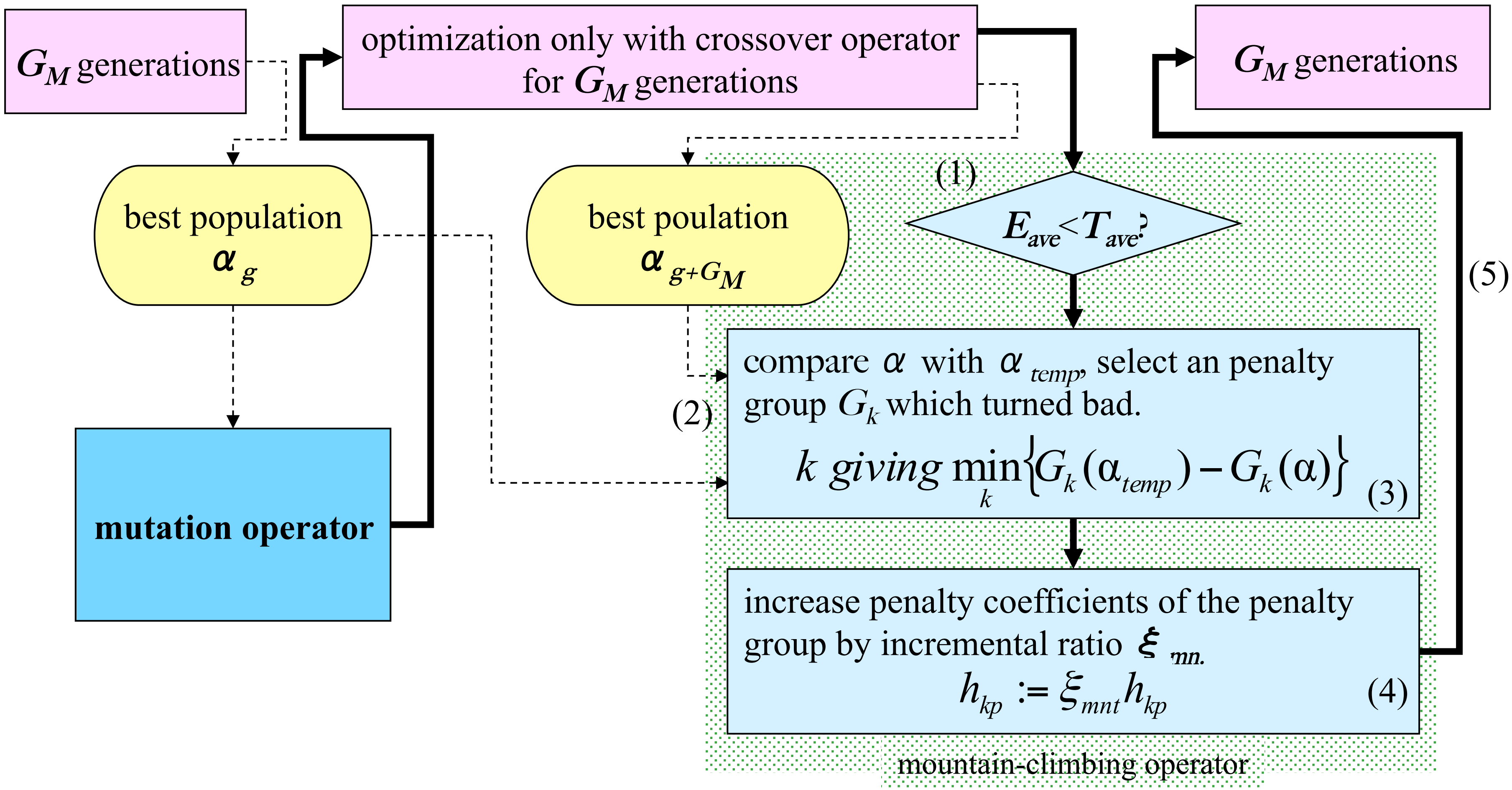
(4) The mutated population is sent to the next G_M generations only with CO.

B. Mountain-Climbing Operator

The optimization of the nurse schedule using the cooperative GA mostly **stagnates** without giving a satisfactory result, even when MO is applied.

The population is caught in a **local minimum area** that both CO and MO cannot escape.

We propose a **Mountain-Climbing Operator** that **increase the penalty coefficients** of the variable penalty function when the optimization **stagnates**.



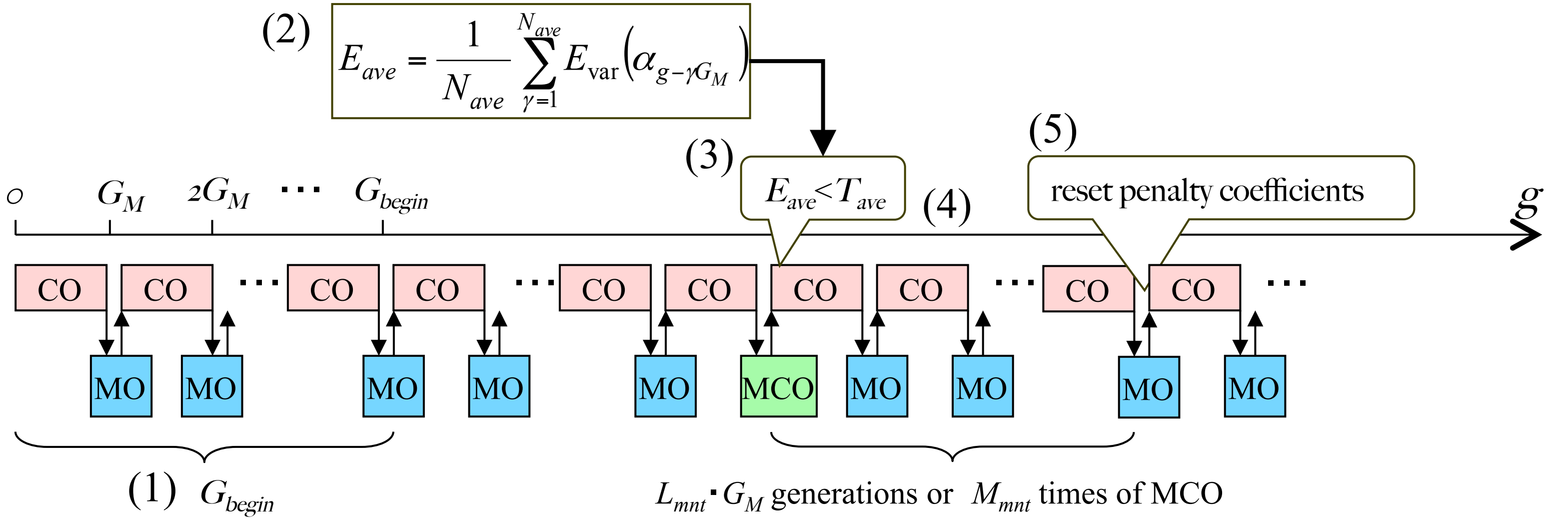
(1) When the average E_{ave} become smaller than the threshold T_{ave}, **MCO** operates in substitution for MO.

(2) MCO **compares** the values of the partial penalty function, G_1-G_4, of the current best population, α_{g+GM}, and those values the previous best population, α_g, respectively.

(3) MCO selects one of the partial penalty groups that gives the **most worsened value**.

(4) The penalty coefficients of the penalty group, k, are increased by a ratio ξ_{mnt}.

(5) Therefore, the penalty function is temporarily modified by MCO.



(1) MCO is available during from G_{begin} generations to G_{end} generations.
(2) Average of the variable penalty function, E_{ave} , for N_{ave} periods is computed.
(3) When the average E_{ave} becomes smaller than the threshold T_{ave}, MCO operates in substitution for MO.
(4) Once MCO operates, it does not operate for more than $N_{ave} \cdot G_M$ generations.
(5) When $L_{mnt} \cdot G_M$ generations have passed or when MCO operates M_{mnt} times, all the penalty coefficients are reset as their default value.

5. Computer Experiments

● Computer experiments are made in order to inspect the effectiveness of our techniques.

- the number of nurses in the section: 23
- the number of duty dates of the current month : 30

Table IV. Parameters for the Crossover, Mutation and Mountain- Climbing Operators.

the number of child pairs of crossover		200
mutation period	G_M	250
the number of period for G_M generations to make average of the penalty value	N_{ave}	5
threshold for starting the mountain climbing	T_{ave}	8.0
incremental ratio	ξ_{mnt}	1.2
the number of period for G_M generations to reset the penalty coefficients	L_{mnt}	20
mountain climbing times to reset the penalty coefficients	M_{mnt}	5

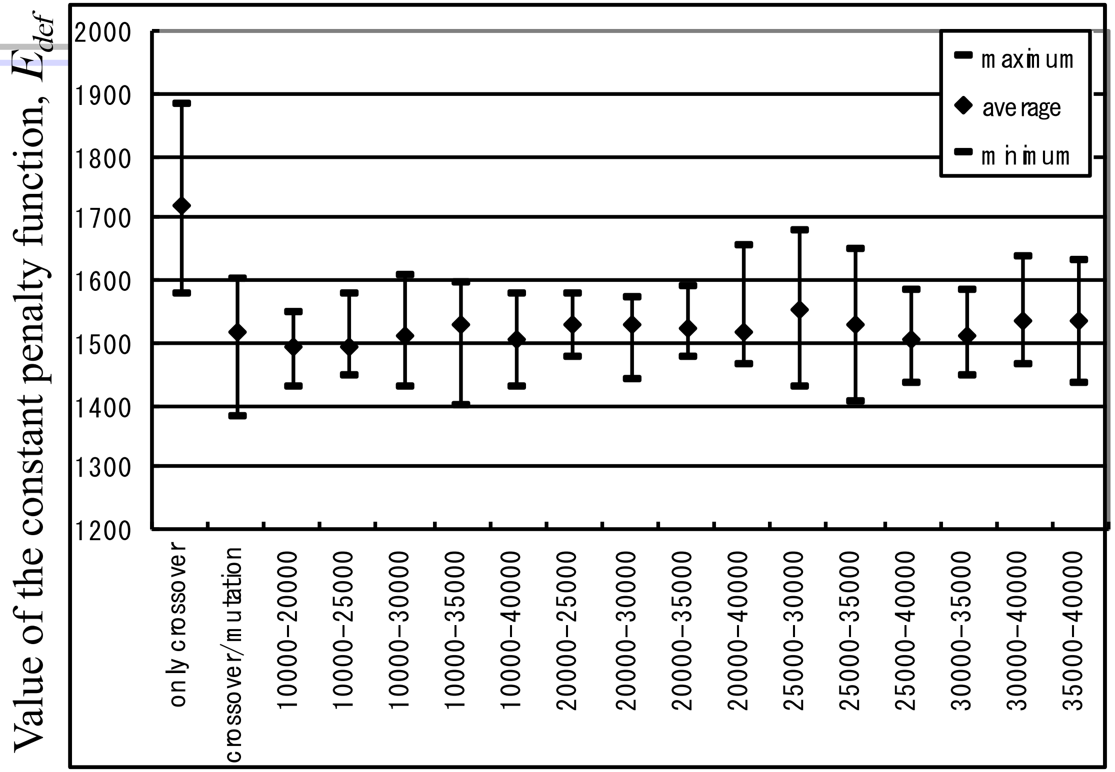


Fig.7. Comparison of optimization results of the nurse scheduling under three types of operator arrangement for **fifty thousands generations**. Pairs of numbers under the horizontal axis denote beginning and finishing generations of MCO.

CO CO+MO CO+MO+MCO

- Ten times trials are executed under each condition.
- The vertical bar shows the MAX. AVERAGE and MIN. values of the constant penalty.
- The results by using CO and MO are comparably **better** than that by using only crossover.
- The results by using CO, MO and MCO are comparably **similar** to that by using CO and MO.

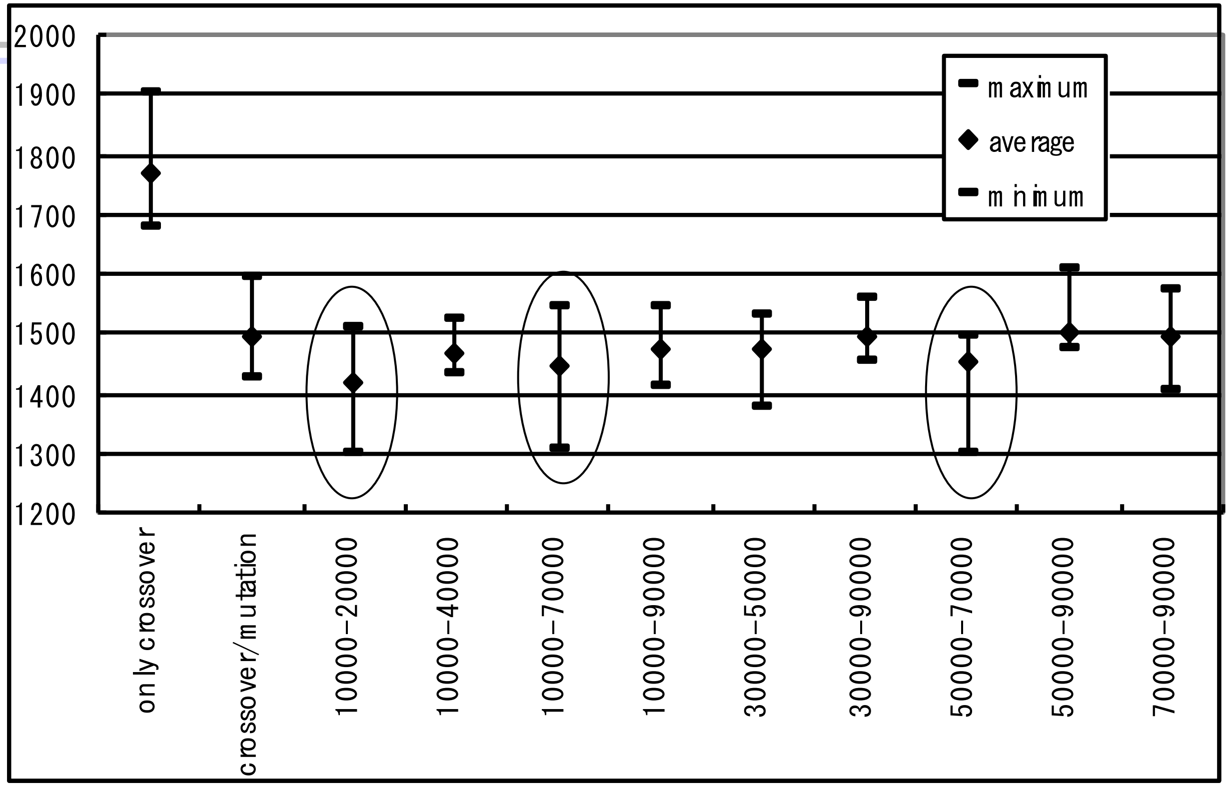


Fig.8. Comparison of optimization results of nurse scheduling by three types of the cooperative GA for **hundred thousands generations**.

- The results by using only **CO** is almost **similar** to that in the case of **fifty thousands generations**.
- The results by using **CO and MO** is also almost **similar** to that in the case of **fifty thousands generations**.

- Some results by using **CO, MO and MCO** give **better** solutions.
- This means that the MCO is effective for the cooperative GA for the nurse scheduling problem.

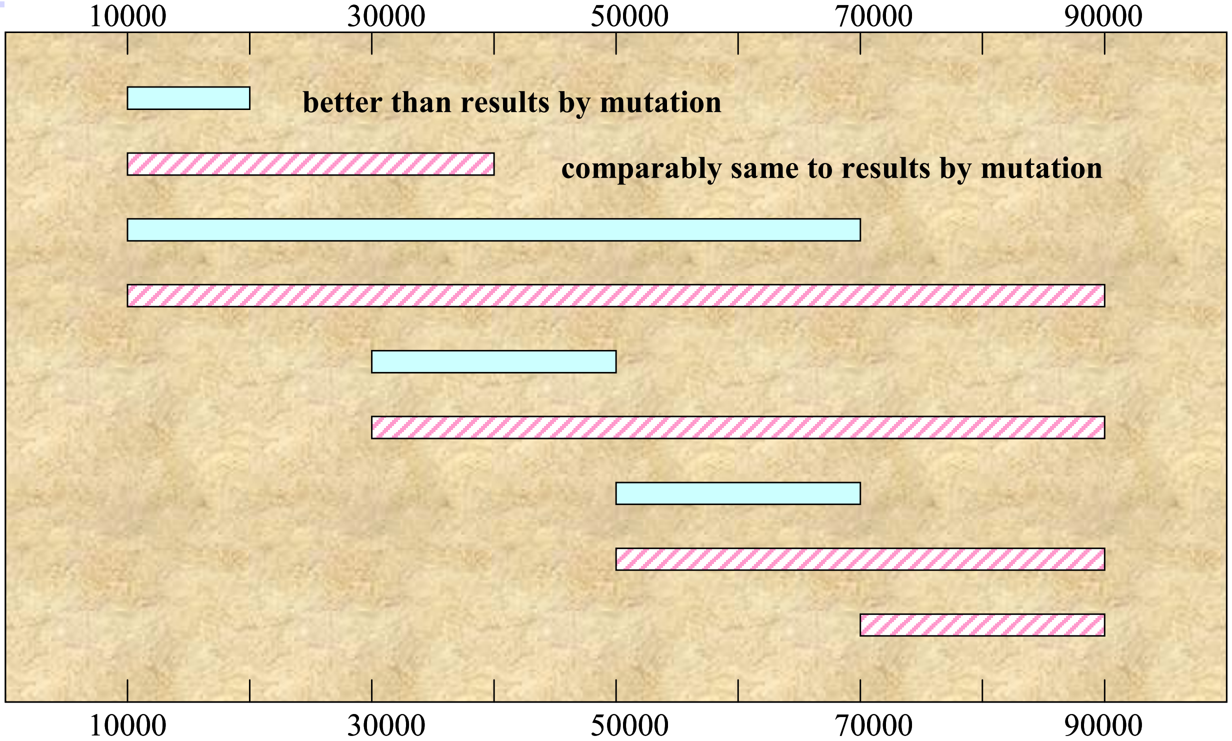


Fig.9 The generation intervals that the mountain climbing operator is applied. The gray intervals show trials giving the better results. And the striped intervals show trials giving almost same results to the case by using CO and MO.

- Finally, we have compared the generation intervals while MCO is available.
- The better results are obtained in the cases when MCO is available for the shorter interval beginning from the earlier generation until the mid-term.

6. Conclusion

We have applied the **cooperative GA** to the **nurse scheduling problem**.

Conventional Way

The cooperative GA optimizes the schedule only by using **CO**.

(∵ **CO** always keeps **consistency** of the schedule.)

In contrast

◆We have proposed **MO** for the cooperative GA.
◆**MO does not obstruct** consistency of the schedule.
◆By means of MO, the cooperative GA can give the **better** schedule.

The cooperative GA with **CO and MO stagnates** in the middle or the end term of the optimization.
Because the optimization is thrown into a large local minimum.

To escape from such a large local minimum

◆We have proposed MCO for the cooperative GA that temporarily modifies the penalty function.
◆By using MCO, the cooperative GA does not stagnate and can globally optimize the nurse schedule.

Problems in the Future

- We will investigate **other different requirements** about the nurse scheduling.

- MCO is hard to apply to the cooperative GA, because the current version of MCO has **many parameters**. We have to consider generalization of those parameters.

- We will develop software using the algorithm that we have proposed in this paper.

Thank you very much for your kind attention!

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